Specializing Scope Graph Resolution Queries

Aron Zwaan

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Overview: Synthesizing Type Checkers

- Implementing type checkers is hard: generate using Statix

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Name & Type Specification

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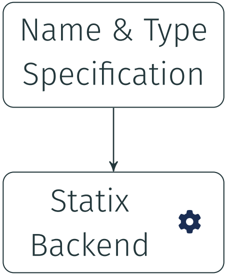

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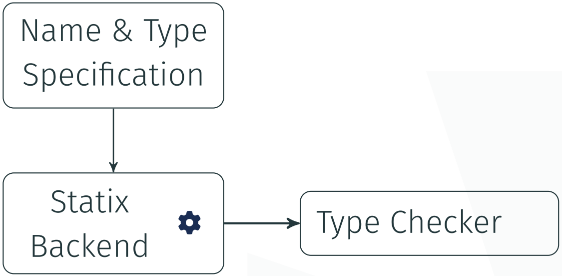

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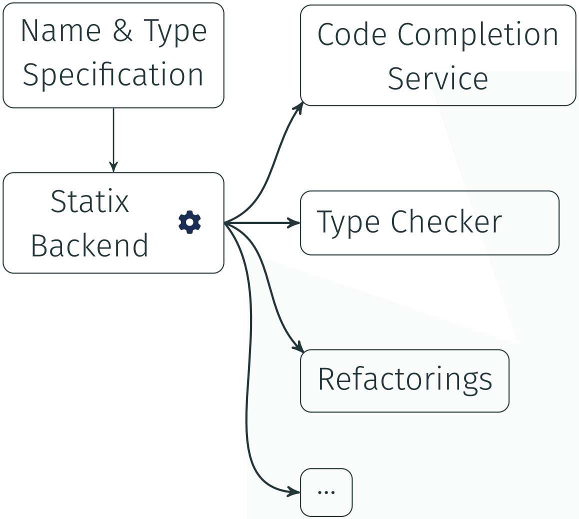

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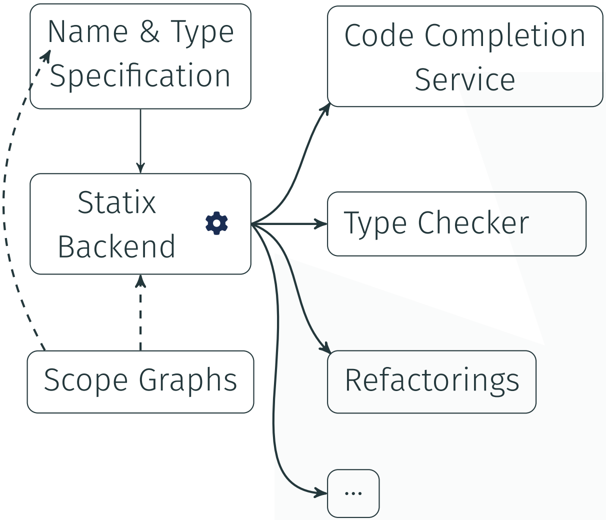

Overview: Synthesizing Type Checkers

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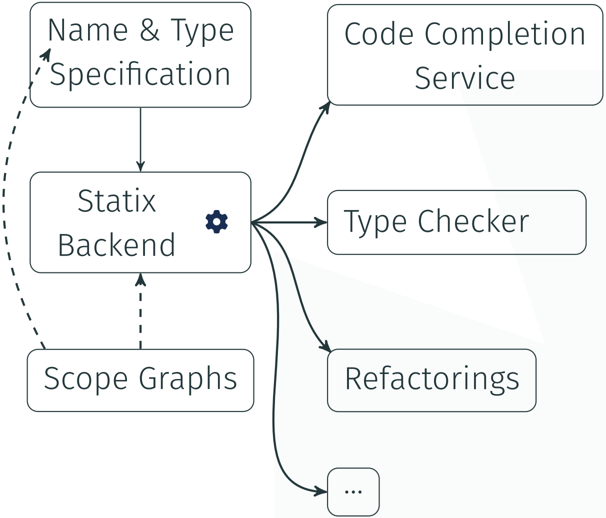
- Advantages of generating
  1. Easy
  2. Consistent
  3. Allows reasoning



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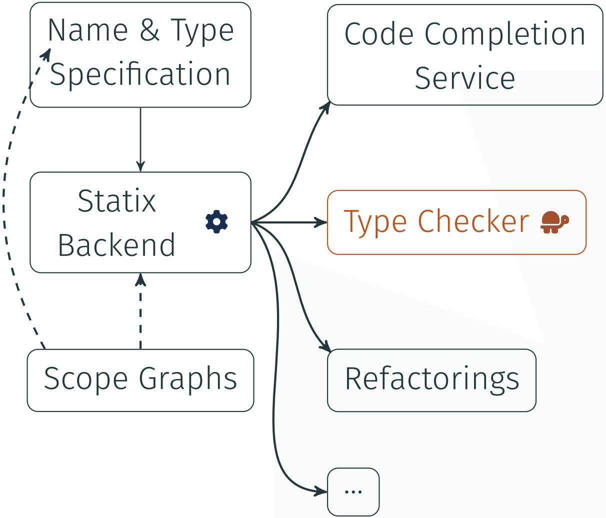

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* Implementing type checkers is hard: generate using Statix
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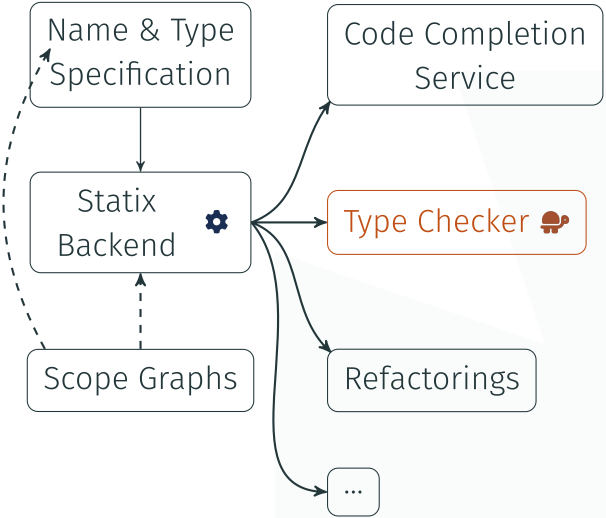
* Problem: 50% overhead in name resolution algorithm
* Solution: partial evaluation



Specializing Scope Graph Resolution Queries

Scope Graphs

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def a = 42;
module A {
  def x = 6
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module B {
  import A;
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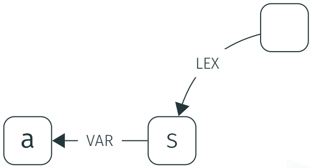

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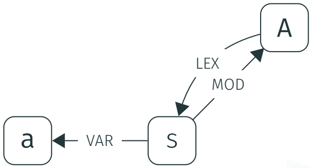

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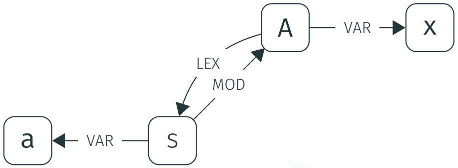

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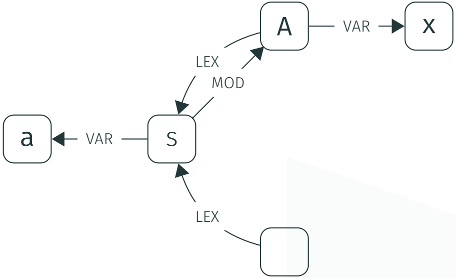

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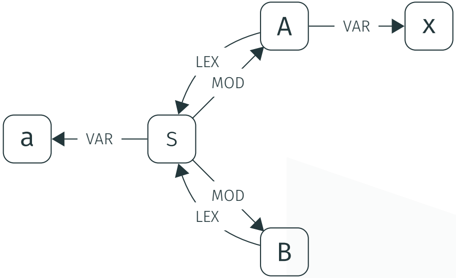

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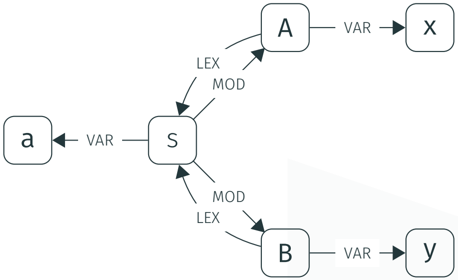

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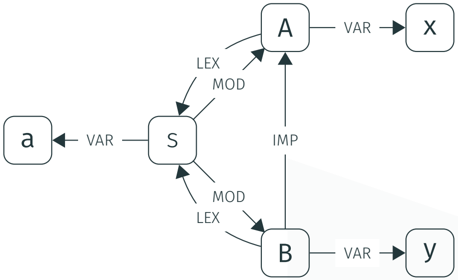

Specializing Scope Graph Resolution Queries

Name Resolution in Scope Graphs: Queries

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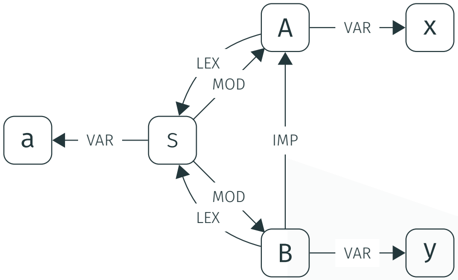

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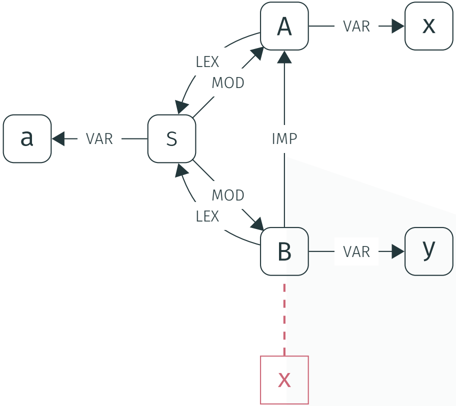

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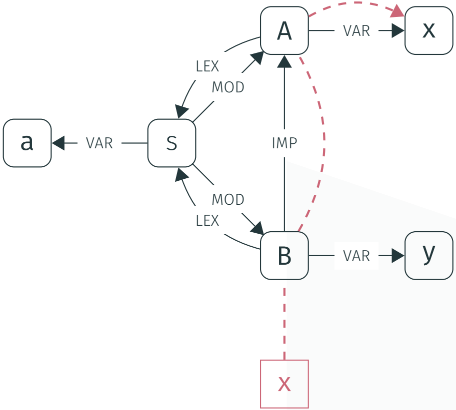

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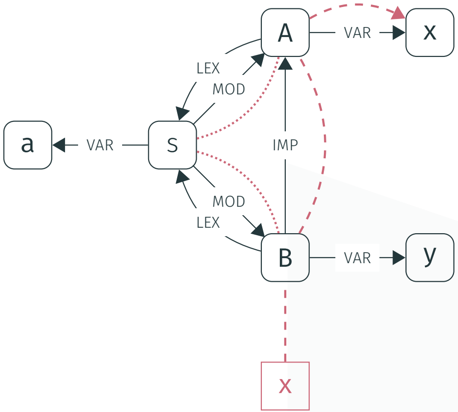

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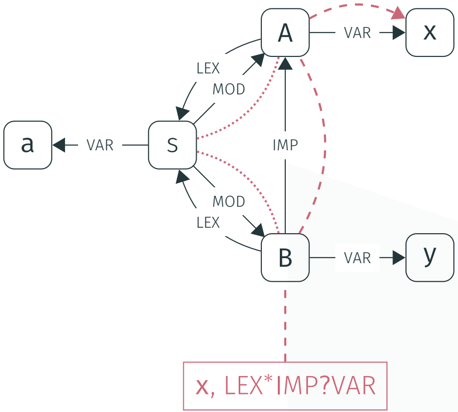

Specializing Scope Graph Resolution Queries

Resolution (simplified)

```
Resolve(𝒢, s, ℛ, n) {
  E := ∅
  if s = n & ϵ ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂_ℓ ℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂_ℓ ℛ, n)
  return E
}
```

Resolution (simplified)

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Resolution (simplified)

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Resolve(𝒢, s, ℛ, n) {
  E := ∅
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  for ℓ in ℒ
    if [∂ℓℛ] ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, [∂ℓℛ], n)
  return E
}
```

- $\partial_\ell \mathcal{R} = \mathcal{R}' \triangleq \ell\omega \in \mathcal{R} \iff \omega \in \mathcal{R}'$
- Examples
  - $\partial_{L_1} L_1 L_2 = L_2$
  - $\partial_{L_2} L_1 ? L_2^+ = L_2^*$
  - $\partial_{L_3} L_1 ? L_2^+ = \varnothing$

Resolution (simplified)

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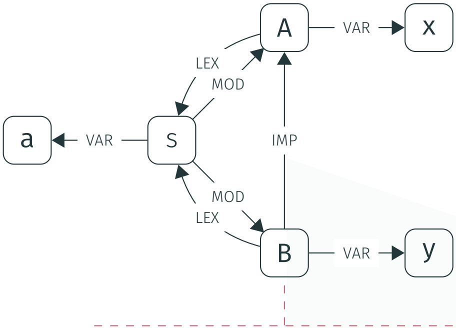

$s = \text{B}, \mathcal{R} = \text{LEX}^*\text{IMP?VAR}, n = \text{x}$

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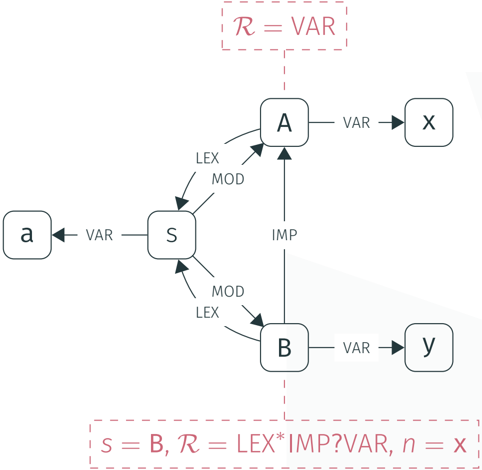

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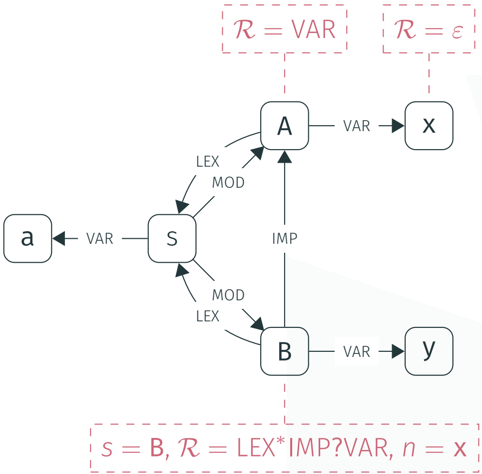
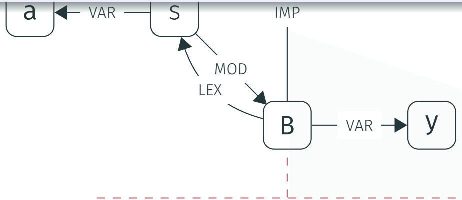


Resolution (simplified)

$\mathcal{R} = \mathsf{VAR}$ $\mathcal{R} = \varepsilon$

Profiling: 12.5% overhead for computing derivatives.

`if` $\partial_\ell \mathcal{R} \neq \varnothing$
  `for` s' `in getEdges(`$\mathcal{G}, s, \ell$`)`
    `E += Resolve(`$\mathcal{G}, s', \partial_\ell \mathcal{R}, n$`)`
`return E`
`}`



$s = \mathsf{B}, \mathcal{R} = \mathsf{LEX}^*\mathsf{IMP}?\mathsf{VAR}, n = \mathsf{x}$

Resolution (simplified)

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  for s' in getEdges($\mathcal{G}, s, \ell$)
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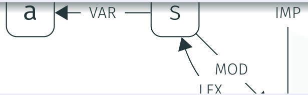

$\mathcal{R}$ known statically: specialize `Resolve`.

$s = \mathsf{B}$, $\mathcal{R} = \mathsf{LEX^*IMP?VAR}$, $n = \mathsf{x}$

Specializing Scope Graph Resolution Queries

Specialization (simplified)

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Resolve(𝒢, s, ε, n) {
  E := ∅
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  for ℓ ∈ ℒ
    if ∂ℓε ≠ ∅
      for s′ ∈ getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓε, n)
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}
```

```
Resolve_ε(𝒢, s, n) {

}
```

Specialization (simplified)

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  for ℓ ∈ ℒ
    if ∂_ℓ ε ≠ ∅
      for s′ ∈ getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂_ℓ ε, n)
  return E
}
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}
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Resolve_ε(𝒢, s, n) {
  E := ∅
  if s = n
    E += { s }
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}
```

Specialization (simplified)

```
Resolve(𝒢, s, LEX*IMP?VAR, n) {
  E := ∅
  if ϵ ∈ LEX*IMP?VAR & s = n
    E += { s }
  for ℓ ∈ ℒ
    if ∂ℓLEX*IMP?VAR ≠ ∅
      for s′ ∈ getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓLEX*IMP?VAR, n)
  return E
}
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Resolve_LEX*IMP?VAR(𝒢, s, n) {

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        E += Resolve(𝒢, s′, ∂ℓLEX*IMP?VAR, n)
  return E
}
```

```
Resolve_LEX*IMP?VAR(𝒢, s, n) {
  E := ∅

}
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Specialization (simplified)

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Resolve(𝒢, s, LEX*IMP?VAR, n) {
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  if ϵ ∈ LEX*IMP?VAR & s = n
    E += { s }
  for ℓ ∈ ℒ
    if ∂ℓLEX*IMP?VAR ≠ ∅
      for s′ ∈ getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓLEX*IMP?VAR, n)
  return E
}
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        E += Resolve(𝒢, s′, ∂ℓLEX*IMP?VAR, n)
  return E
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Resolve_LEX*IMP?VAR(𝒢, s, n) {
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Specialization (simplified)

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Resolve(𝒢, s, LEX*IMP?VAR, n) {
  E := ∅
  if ϵ ∈ LEX*IMP?VAR & s = n
    E += { s }
  for ℓ ∈ ℒ                                  ℓ = VAR
    if ∂ℓLEX*IMP?VAR ≠ ∅
      for s′ ∈ getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓLEX*IMP?VAR, n)
  return E
}
```

```
Resolve_LEX*IMP?VAR(𝒢, s, n) {
  E := ∅
  for s′ ∈ getEdges(𝒢, s, VAR)
    E += Resolve_ε(𝒢, s′, n)
}
```

Specialization (simplified)

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Resolve(𝒢, s, LEX*IMP?VAR, n) {
  E := ∅
  if ϵ ∈ LEX*IMP?VAR & s = n
    E += { s }
  for ℓ ∈ ℒ                                    ℓ = LEX
    if ∂ℓLEX*IMP?VAR ≠ ∅
      for s′ ∈ getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓLEX*IMP?VAR, n)
  return E
}
```

```
Resolve_LEX*IMP?VAR(𝒢, s, n) {
  E := ∅
  for s′ ∈ getEdges(𝒢, s, VAR)
    E += Resolve_ε(𝒢, s′, n)
  for s′ ∈ getEdges(𝒢, s, LEX)
    E += Resolve_LEX*IMP?VAR(𝒢, s′, n)
}
```

Specialization (simplified)

```
Resolve(𝒢, s, LEX*IMP?VAR, n) {
  E := ∅
  if ϵ ∈ LEX*IMP?VAR & s = n
    E += { s }
  for ℓ ∈ ℒ                                    ℓ = IMP
    if ∂ℓLEX*IMP?VAR ≠ ∅
      for s′ ∈ getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓLEX*IMP?VAR, n)
  return E
}
```

```
Resolve_LEX*IMP?VAR(𝒢, s, n) {
  E := ∅
  for s′ ∈ getEdges(𝒢, s, VAR)
    E += Resolve_ε(𝒢, s′, n)
  for s′ ∈ getEdges(𝒢, s, LEX)
    E += Resolve_LEX*IMP?VAR(𝒢, s′, n)
  for s′ ∈ getEdges(𝒢, s, IMP)
    E += Resolve_VAR(𝒢, s′, n)

}
```

Specialization (simplified)

```
Resolve(𝒢, s, LEX*IMP?VAR, n) {
  E := ∅
  if ϵ ∈ LEX*IMP?VAR & s = n
    E += { s }
  for ℓ ∈ ℒ                                   ℓ = MOD
    if ∂ℓLEX*IMP?VAR ≠ ∅
      for s′ ∈ getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓLEX*IMP?VAR, n)
  return E
}
```

```
Resolve_LEX*IMP?VAR(𝒢, s, n) {
  E := ∅
  for s′ ∈ getEdges(𝒢, s, VAR)
    E += Resolve_ε(𝒢, s′, n)
  for s′ ∈ getEdges(𝒢, s, LEX)
    E += Resolve_LEX*IMP?VAR(𝒢, s′, n)
  for s′ ∈ getEdges(𝒢, s, IMP)
    E += Resolve_VAR(𝒢, s′, n)

}
```

Specialization (simplified)

```
Resolve(𝒢, s, LEX*IMP?VAR, n) {
  E := ∅
  if ϵ ∈ LEX*IMP?VAR & s = n
    E += { s }
  for ℓ ∈ ℒ
    if ∂ℓ LEX*IMP?VAR ≠ ∅
      for s′ ∈ getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓ LEX*IMP?VAR, n)
  return E
}
```

```
Resolve_LEX*IMP?VAR(𝒢, s, n) {
  E := ∅
  for s′ ∈ getEdges(𝒢, s, VAR)
    E += Resolve_ε(𝒢, s′, n)
  for s′ ∈ getEdges(𝒢, s, LEX)
    E += Resolve_LEX*IMP?VAR(𝒢, s′, n)
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    E += Resolve_VAR(𝒢, s′, n)

}
```

Specialization (simplified)

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Resolve(𝒢, s, LEX*IMP?VAR, n) {
  E := ∅
  if ϵ ∈ LEX*IMP?VAR & s = n
    E += { s }
  for ℓ ∈ ℒ
    if ∂ℓLEX*IMP?VAR ≠ ∅
      for s′ ∈ getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓLEX*IMP?VAR, n)
  return E
}
```

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Resolve_LEX*IMP?VAR(𝒢, s, n) {
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  for s′ ∈ getEdges(𝒢, s, VAR)
    E += Resolve_ε(𝒢, s′, n)
  for s′ ∈ getEdges(𝒢, s, LEX)
    E += Resolve_LEX*IMP?VAR(𝒢, s′, n)
  for s′ ∈ getEdges(𝒢, s, IMP)
    E += Resolve_VAR(𝒢, s′, n)
  return E
}
```

Summary

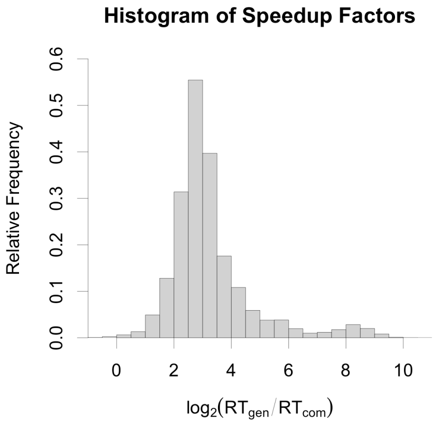
Also in the paper

- Encoding shadowing using partial order on labels
- Resolution algorithm *with shadowing*
- Shadowing: 35% overhead
- Specializing queries shadowing
- Optimizations on IR

Evaluation: Setup

- Java Specification
- Apache Commons CSV, IO and Lang3 projects
- Micro-benchmarks: Speedup of individual queries (CSV)
- Macro-benchmarks: Speedup of type checkers

Evaluation: Results



Project	#Queries	$RT_{gen}(s)$	$RT_{com}(s)$	Speedup
CSV	14328	7.3	4.5	39%
IO	73843	19	12	38%
Lang3	288883	88	46	48%

Bigger Picture

* What made this work so well?
  1. Complex algorithm to interpret language construct (`Resolve`)
  2. Statically known parameters
* Common for more declarative languages

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  1. Complex algorithm to interpret language construct (`Resolve`)
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* Common for more declarative languages

Suggests specialization is especially powerful for declarative languages.

Conclusion

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Scope graphs allow declarative but executable specification of name resolution.

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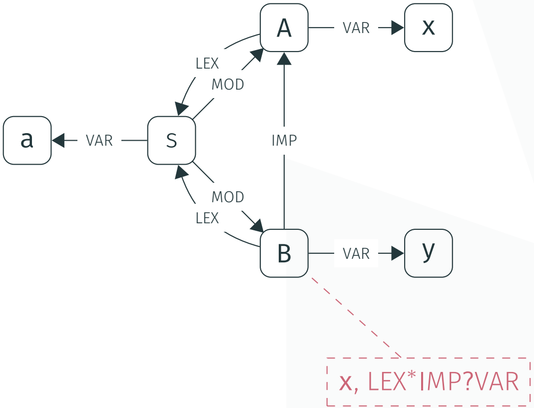
Scope graphs allow declarative but executable specification of name resolution.

Specialization especially speeds up interpreters of declarative languages.

Resolution (simplified)

```
Resolve(𝒢, s, ℛ, n) {
  E := ∅
  if s = n & ϵ ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```



Resolution (simplified)

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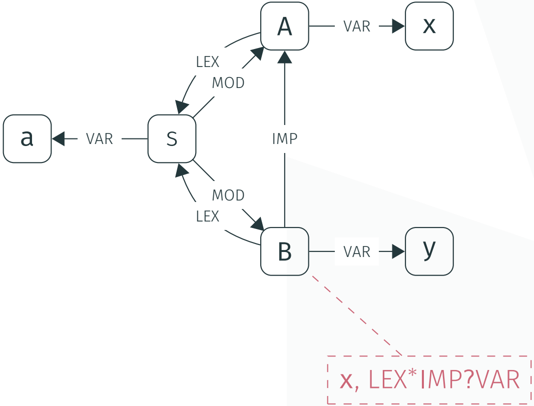
Resolve(𝒢, s, ℛ, n) {
  E := ∅
  if s = n & ϵ ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂_ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂_ℓℛ, n)
  return E
}
```



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```
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    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
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}
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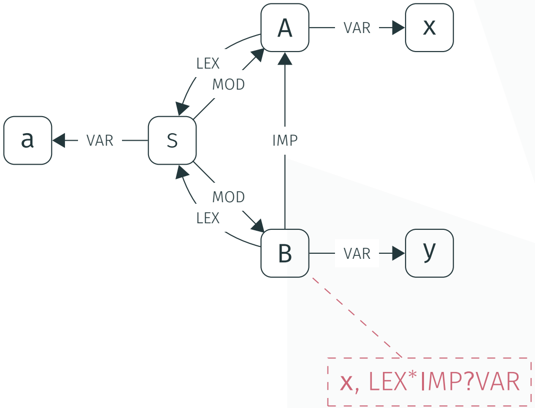

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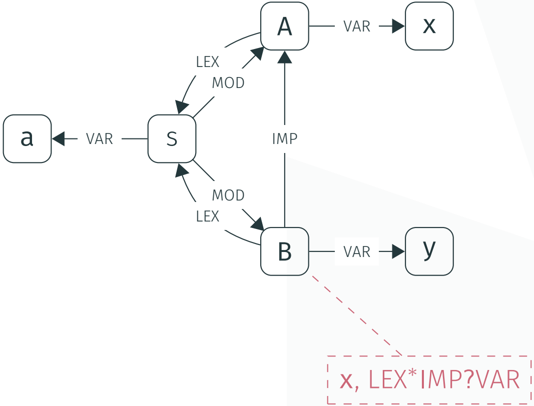

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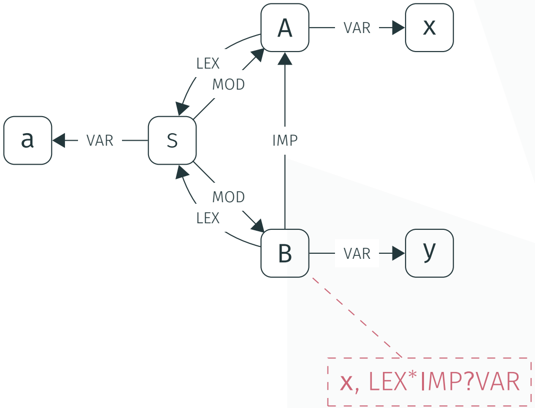
}
```



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        E += Resolve(𝒢, s′, ∂_ℓℛ, n)
  return E
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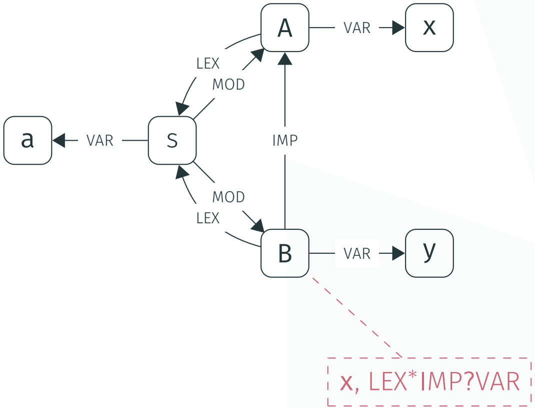

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  return E
}
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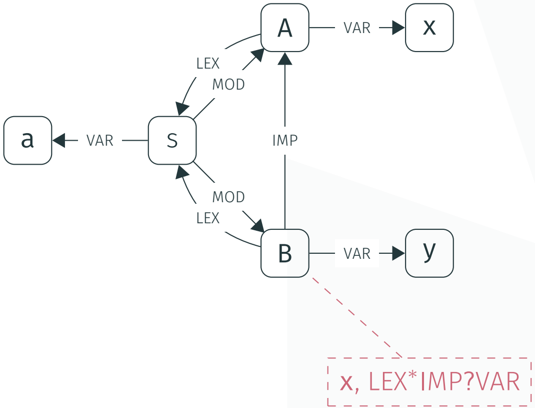

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        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```

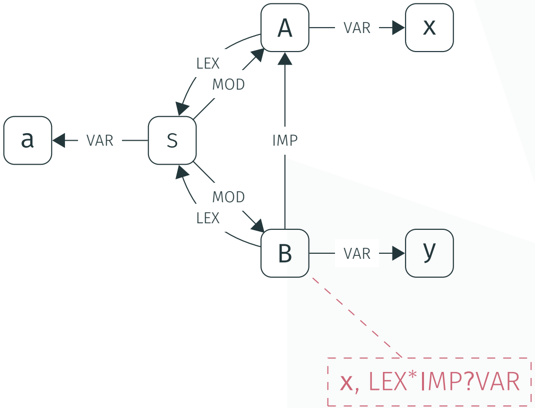

Resolution (simplified)

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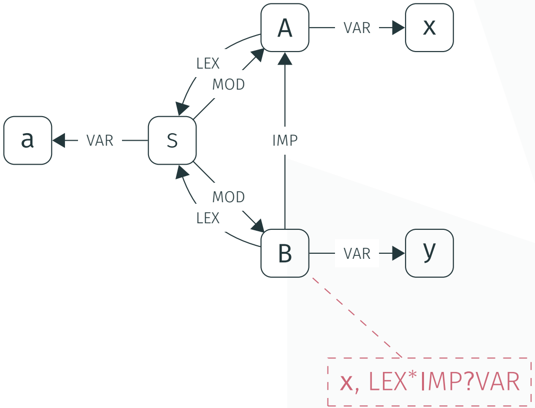
    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```



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        E += Resolve(𝒢, s′, ∂ℓℛ, n)
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}
```

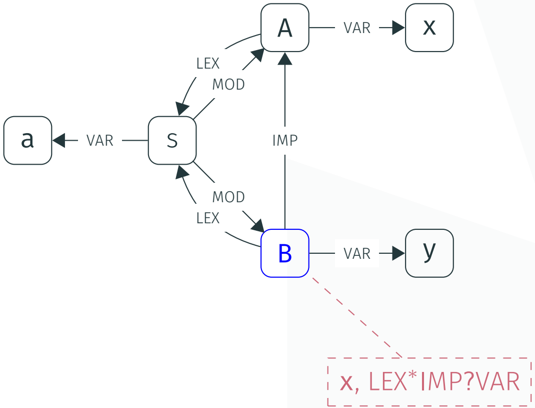

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```
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  E := ∅
  if s = n & ϵ ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂_ℓ ℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂_ℓ ℛ, n)
  return E
}
```
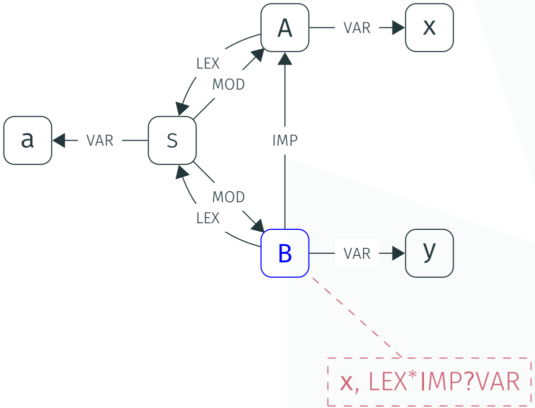
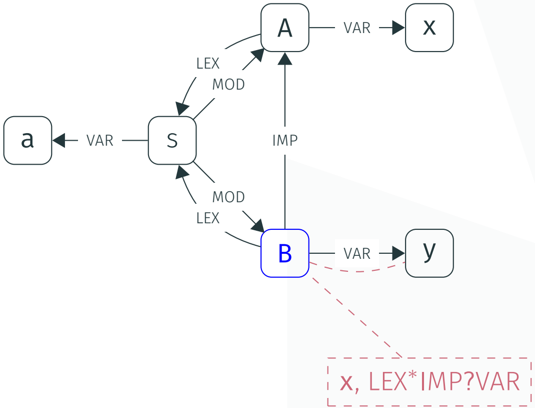

$$\partial_\ell \mathcal{R} = \mathcal{R}' \triangleq \ell\omega \in \mathcal{R} \iff \omega \in \mathcal{R}'$$

E.g.: $\partial_{L_1} L_1?L_2^* = L_2^*$



Resolution (simplified)

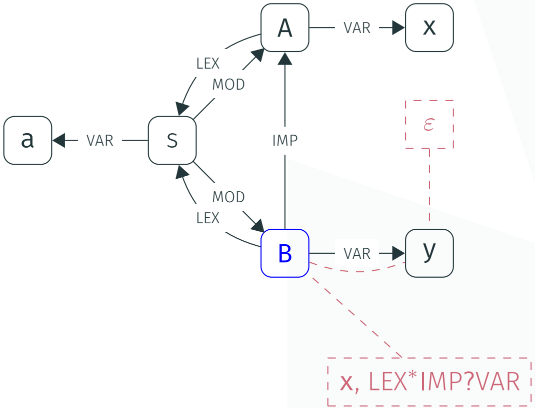
```
Resolve(𝒢, s, ℛ, n) {
  E := ∅
  if s = n & ε ∈ ℛ
    E += { s }
  for ℓ in ℒ                                ℓ = VAR
    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```



Resolution (simplified)

```
Resolve(𝒢, s, ℛ, n) {
  E := ∅
  if s = n & ϵ ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```

$\ell = \text{VAR}$



Resolution (simplified)

```
Resolve(𝒢, s, ℛ, n) {
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  if s = n & ϵ ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```

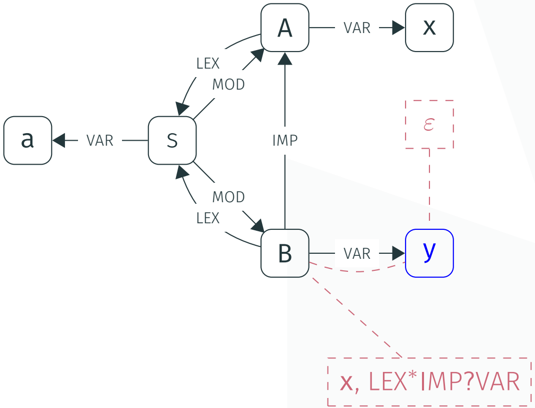

Resolution (simplified)

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Resolve(𝒢, s, ℛ, n) {
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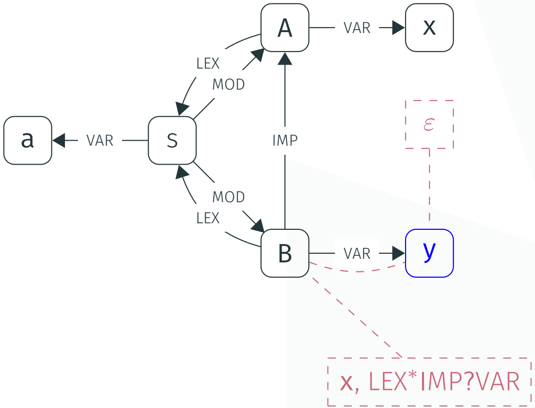
  if s = n & ϵ ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```



Resolution (simplified)

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  E := ∅
  if s = n & ϵ ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E      E = ∅
}
```
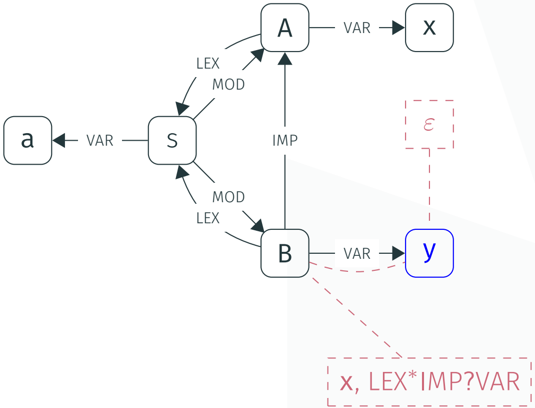
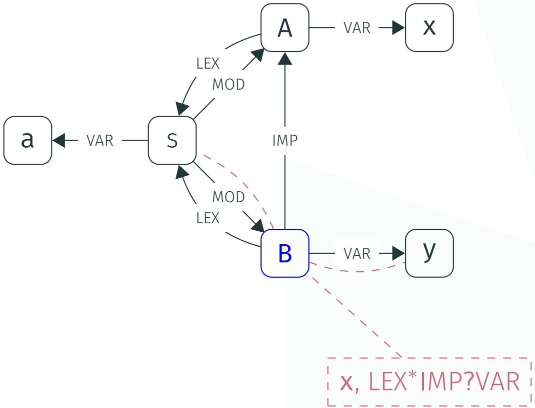



Resolution (simplified)

```
Resolve(𝒢, s, ℛ, n) {
  E := ∅
  if s = n & ϵ ∈ ℛ
    E += { s }
  for ℓ in ℒ                                ℓ = LEX
    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```



Resolution (simplified)

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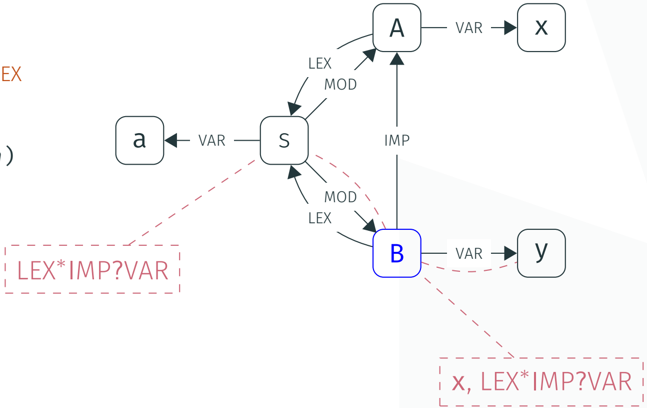
  E := ∅
  if s = n & ϵ ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```

$\ell = \text{LEX}$



Resolution (simplified)

```
Resolve(𝒢, s, ℛ, n) {
  E := ∅
  if s = n & ε ∈ ℛ
    E += { s }
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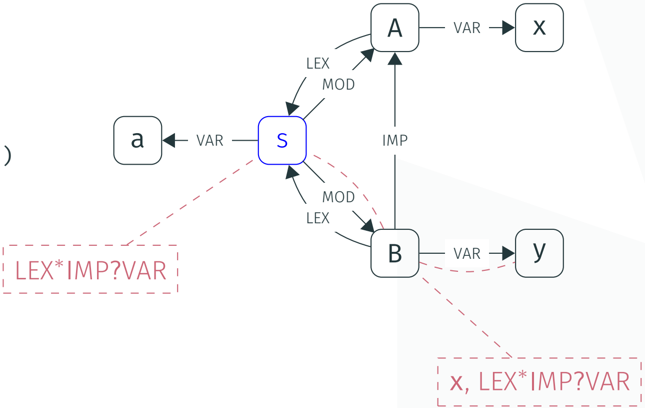
    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```



Resolution (simplified)

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Resolve(𝒢, s, ℛ, n) {
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    E += { s }
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    if ∂ℓℛ ≠ ∅
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        E += Resolve(𝒢, s′, ∂ℓℛ, n)
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}
```

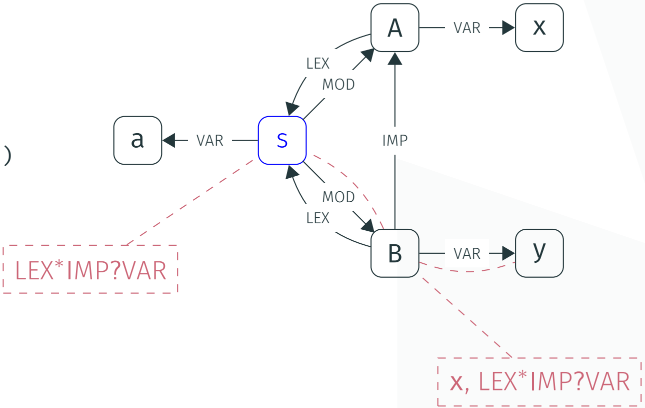

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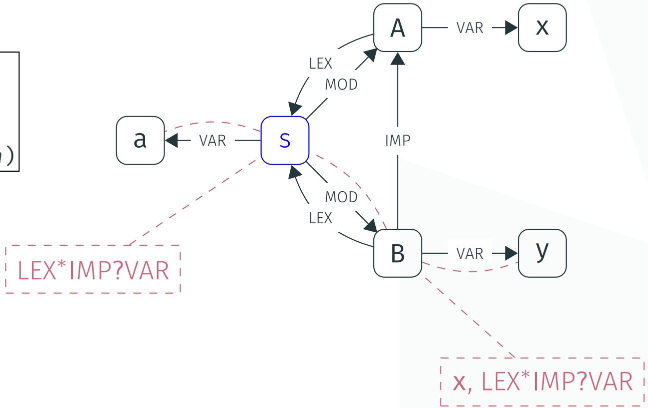
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```



Resolution (simplified)

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Resolve(𝒢, s, ℛ, n) {
  E := ∅
  if s = n & ϵ ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂_ℓ ℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂_ℓ ℛ, n)
  return E
}
```
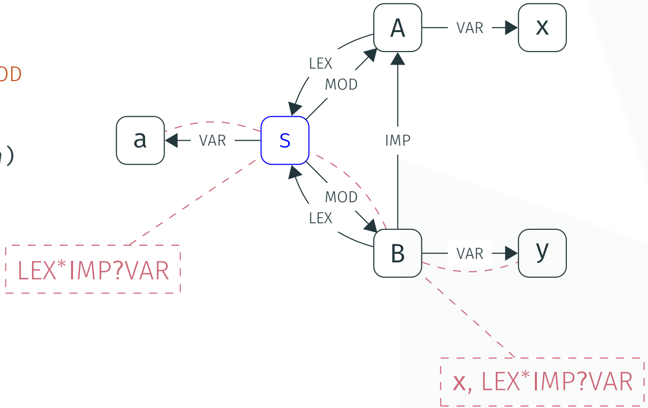

$\ell = \text{MOD}$



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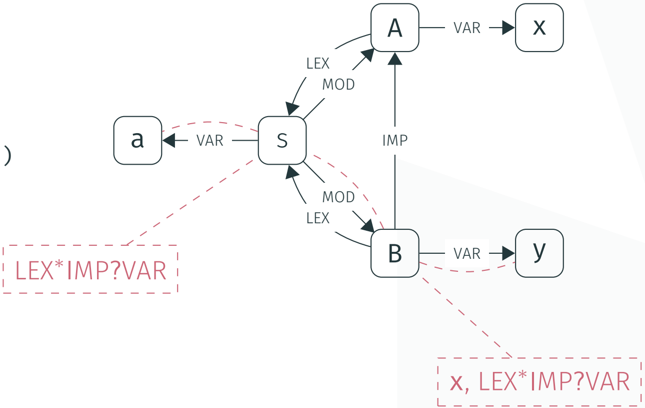
  for ℓ in ℒ
    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```

$E = \emptyset$



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        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```
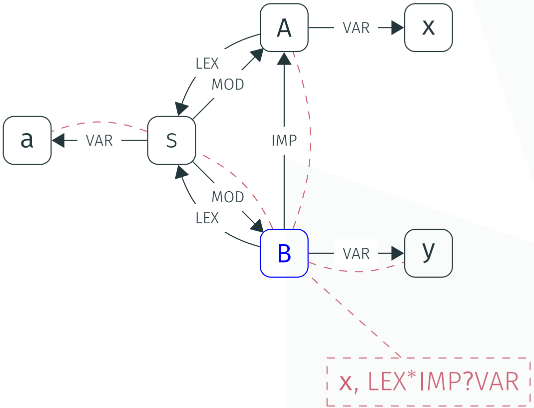
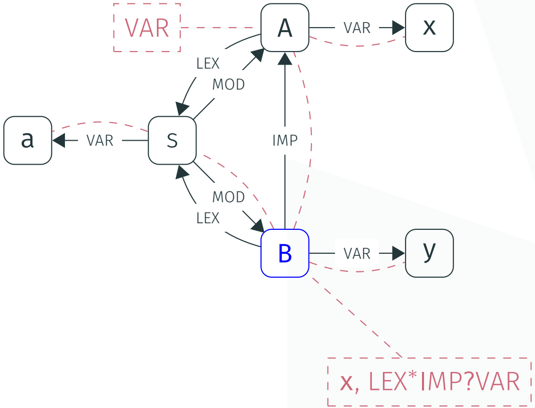

$\ell = \text{IMP}$



Resolution (simplified)

```
Resolve(𝒢, s, ℛ, n) {
  E := ∅
  if s = n & ε ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, [∂ℓℛ], n)
  return E
}
```

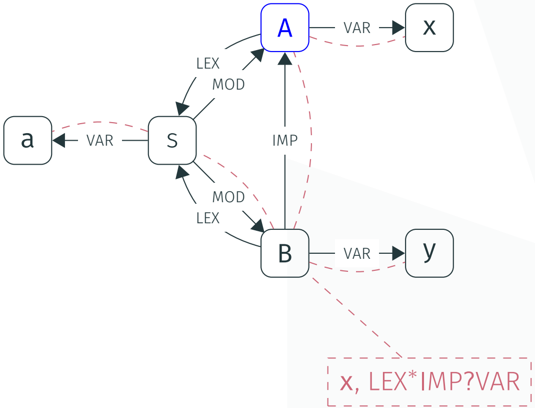
$\ell = \text{IMP}$



Resolution (simplified)

```
Resolve(𝒢, s, ℛ, n) {
  E := ∅
  if s = n & ϵ ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```

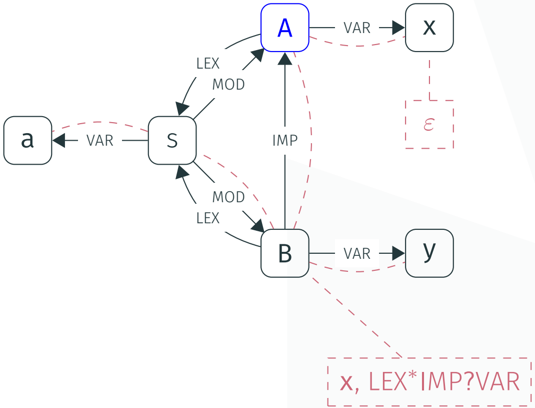
$\ell = \text{VAR}$



Resolution (simplified)

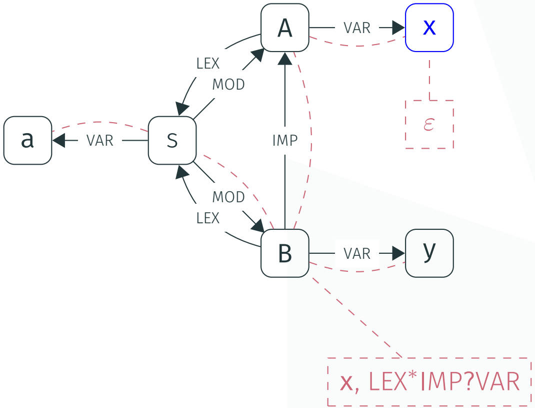
```
Resolve(𝒢, s, ℛ, n) {
  E := ∅
  if s = n & ϵ ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```

$\ell = \text{VAR}$



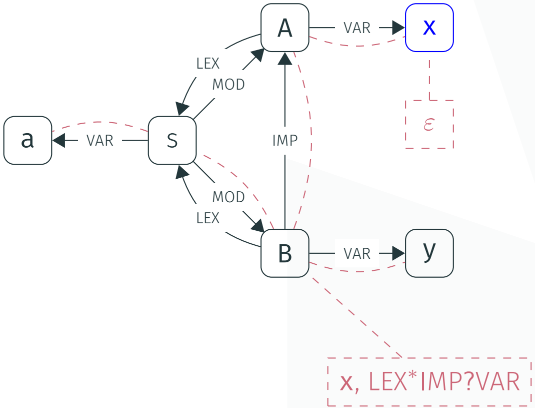
Resolution (simplified)

```
Resolve(𝒢, s, ℛ, n) {
  E := ∅
  if s = n & ϵ ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂_ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂_ℓℛ, n)
  return E
}
```



Resolution (simplified)

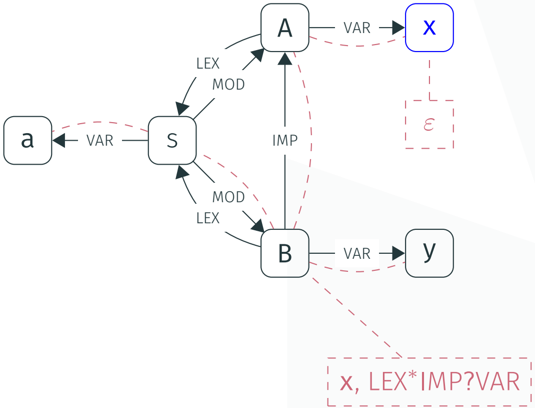
```
Resolve(𝒢, s, ℛ, n) {
  E := ∅
  if s = n & ϵ ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```



Resolution (simplified)

```
Resolve(𝒢, s, ℛ, n) {
  E := ∅
  if s = n & ε ∈ ℛ
    E += { s }
  for ℓ in ℒ
    if ∂_ℓℛ ≠ ∅
      for s′ in getEdges(𝒢, s, ℓ)
        E += Resolve(𝒢, s′, ∂_ℓℛ, n)
  return E
}
```

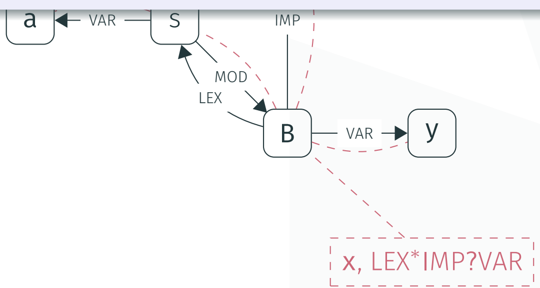
$A = \{ x \}$



Resolution (simplified)

```
Resolve(𝒢,s,ℛ,n) {
    for s′ in getEdges(𝒢,s,ℓ)
        E += Resolve(𝒢,s′,∂ℓℛ,n)
  return E
}
```

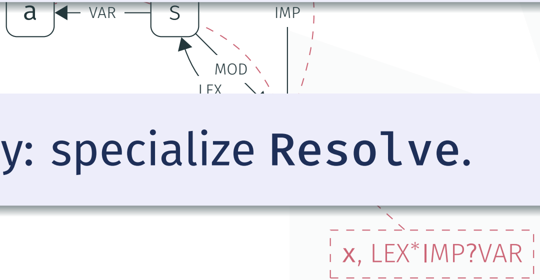
Profiling: 12.5% overhead for computing derivatives.



Resolution (simplified)

```
Resolve(𝒢, s, ℛ, n) {
      E += Resolve(𝒢, s′, ∂ℓℛ, n)
  return E
}
```

Profiling: 12.5% overhead for computing derivatives.



$\mathcal{R}$ known statically: specialize `Resolve`.